Using Technology to Integrate Geometry and Algebra in the Study of Functions

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Report

This group discussed the potential value of using technology-supported geometric transformations to introduce and develop function concepts. This approach (referred to here as *Geometric Functions*) and related representations can be used to help students develop intuitive understandings, avoid and overcome misconceptions, and deepen their understanding of variables and functions.

Why is this approach not more widely used? What are the benefits and obstacles? How can this approach be encouraged and facilitated? The session agendas and notes are available on the DG9 wiki, which also contains links to resources (including movies and existing student activities): http://wiki.geometricfunctions.com/index.php/ICME_12_Discussion_Group_9.

Our DG addressed a number of questions. Why are geometric transformations not more widely integrated into the study of function? What are the benefits, and what are the obstacles? What experiences have discussants had in promoting such an approach? How can we best encourage and facilitate such a change in students' experience of function?

Each session began with a whole-group introduction, broke into small-group discussions addressing particular areas, and concluded with a whole-group summary. The bullet points below are based on the reports from the small groups.

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S.J. Cho (ed.), *The Proceedings of the 12th International Congress on Mathematical Education*, DOI 10.1007/978-3-319-12688-3_71

What We Know

- Students need to experience a variety of functions to form a robust conception.
- Though other examples may be given, the conventional approach quickly settles down to $\mathcal{R} \rightarrow \mathcal{R}$ functions defined by equations. But many important functions do not merely map real numbers to real numbers.
- The Geometric Functions approach may contribute to a richer concept of function.
- Transforming points versus transforming shapes is an issue; we need to be clear about this distinction.
- We have anecdotal evidence that teachers don't connect algebra and geometry. In most (all?) countries geometric transformations are taught independently of functions. (One participant observed that five mathematics educators from five different countries agreed that geometric transformations are an independent topic from functions in their mathematics curriculum.)
- Students have difficulties with many function-related concepts (variable, function, domain, range, relative rate of change, composition, and inverses).

Research Questions

- How does the Geometric Functions approach differ from current practice? What might it add? Might important elements be lost?
- How can Geometric Functions expand students' understanding of function? How might students' conceptions of variable, function, domain, etc. be strengthened?
- How might this approach help students' concept of function move along the action-process-object (APOS) sequence?
- How does current thinking on embodied cognition support the Geometric Functions approach? How do students experience Geometric Functions as embodied?
- It's important to present functions in a way that does not introduce misconceptions. What impact does this approach have on common misconceptions about function?
- How can physical activities supplement technology-based activities?

Implementation Issues

- When the teacher starts using Geometric Functions activities, how quickly should she go to technology? Can she use some real-world activities before using virtual activities? (See the Function Dance activity, www.geometricfunctions.org/function_dances.html, for one example of this real to virtual transition.)

- Transformations are sometimes taught in the elementary curriculum. How does this affect the use of Geometric Functions in teaching transformations?
- What would teachers need to know about math that may be unfamiliar to them?
- Professional development and support for teachers should be just-in-time.
- Team teaching may be very useful when teaching unfamiliar topics.
- How could teachers get comfortable with the technology? Ideally the technology should be transparent, so that the focus is on the math. Students' experiences should be mathematical rather than magical.
- The goal of experiences with Geometric Functions is to facilitate conversations about what functions really are, and about the connections between Geometric Functions and functions that are normally studied ($\mathcal{R} \rightarrow \mathcal{R}$ functions expressed as equations).
- Assessment is a problem, since students often do not have the opportunity to use technology during tests. How can this situation be corrected?

Conclusion

Discussion Group 9 concluded that teaching geometric transformations as functions has significant potential for improving students' understanding of function concepts and for avoiding common misconceptions, and that dynamic mathematics technology is a promising way for students to experience geometric transformation as a conceptual metaphor on which to ground their conception of function.

DG9 further concluded that more research should be done to establish the benefits of the Geometric Functions approach and to determine effective ways to implement it. Geometric Functions challenge both wide-spread curricular assumptions (that functions belong to algebra, not geometry) and teachers' typical mathematical background and knowledge, and require careful thought and preparation for effective implementation.

Given the poor student understanding of function concepts that results from current practices, and the proven value of incorporating students' sensory-motor systems in the learning process, we encourage mathematics educators and education researchers to take seriously these twin arguments for studying and implementing the Geometric Functions approach.